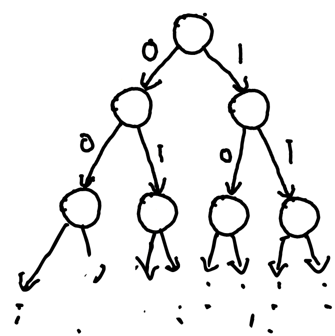
Automorphisms of the Brin–Thompson groups nV

$$\mathrm{Out}(V) \wr \mathrm{Sym}(n) \cong \mathrm{Out}(nV)$$

Cantor Space

$$\mathcal{C}_2 := \{0,1\}^{\omega} \left(= \{ a_0 a_1 a_2 \ldots \mid a_i \in \{0,1\} \ \forall i \in \mathbb{N}_0 \} \right)$$

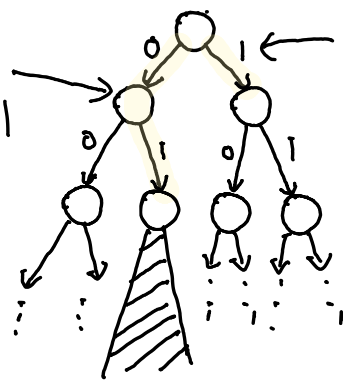


Cantor Space

The finite words correspond to finite downward paths from the root, and the cone at a word is the collection of infinite paths begining with a particular finite path.

the word 01

the word 1

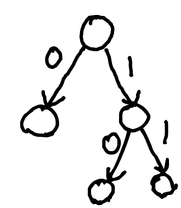


the cone at 01.

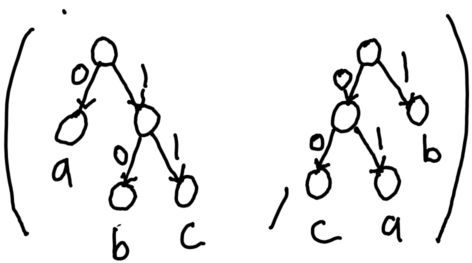
Prefix Codes

A prefix Code is a collection of words whose cones form a partition of $\mathfrak{C}_2$.

The prefix code $\{0, 10, 11\}$



Thompson's Group V (IV)



$0 \mapsto 01$

$10 \mapsto 1$

$11 \mapsto 00$

$011001\ldots \mapsto 0111001\ldots$

$100101\ldots \mapsto 10101\ldots$

$110100\ldots \mapsto 000100\ldots$

Theorem (Rubin)

If G is a "nice" group of homeomorphisms of a "nice" space X then

$$\mathrm{Aut}(G) \cong N_{\mathrm{Homeo}(X)}(G)$$

Corollary

$$\mathrm{Aut}(nV) \cong N_{\mathrm{Homeo}(X)}(nV)$$

<u>Thm</u> (1-D case: Grigorchuk, Nekrshevich, Sushchanskii)
Every continuous function on $\hat{\mathfrak{C}}_2^n$ is transducerable.

<u>Thm</u> (1-D case: Bleak, Cameron, Maissel, Navas, Olukoya)
The transducers for $N_{H(\hat{\mathfrak{C}}_2^n)}(nV)$ are really nice.

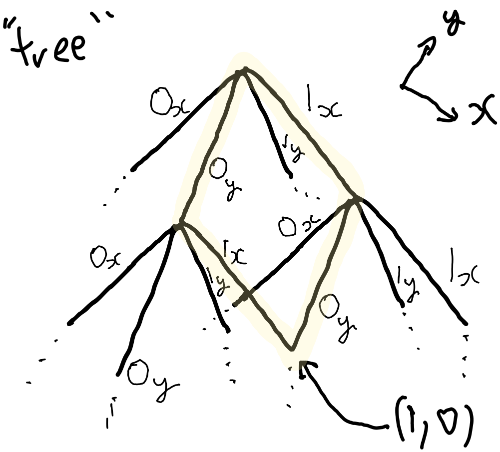
2-dimensional Cantor space $\mathcal{C}_2^2$

for $w \in (\{0,1\}^*)^2$ we define the <u>cone</u> at $w = (u,v) = (u_0 u_1 \dots u_k,\ v_0 v_1 \dots v_l)$ by

$$w\,\mathcal{C}_2^2 := u\,\mathcal{C}_2 \times v\,\mathcal{C}_2$$

$$= \{(u_0 u_1 \dots u_k a_0 a_1 \dots,\ v_0 v_1 \dots v_l b_0 b_1 \dots) \mid a_i, b_i \in \{0,1\}\ \forall i \in \mathbb{N}_0\}.$$

$\mathcal{C}_2^2$ "tree"



$0_x = 0$ in first coordinate
$1_x = 1$ in first coordinate
$0_y = 0$ in second coordinate
$1_y = 1$ in second coordinate

tree $\to$ lattice

Lemma for all $(f_0, f_1, \ldots, f_{n-1}) \in N_{H(\hat{\mathbb{C}}_2)}(V)^n$ the map $f_0 \oplus f_1 \oplus \cdots \oplus f_{n-1} \in N_{H(\hat{\mathbb{C}}_2^n)}(nV)$

Lemma for all $p \in Sym(n)$, the homeomorphism of $\hat{\mathbb{C}}_2^n$ defined by permuting coordinates is in $N_{H(\hat{\mathbb{C}}_2^n)}(nV)$.

Cor $Aut(V) \wr Sym(n) \hookrightarrow Aut(nV)$. (not epic)

Thm $Out(nV) \cong Out(V) \wr Sym(n)$.